$m=0$

$$\frac{1}{N}<\epsilon$$

For $n>N$ $\quad |a_n - a_0| < \frac{1}{N} < \epsilon$

$$\Rightarrow \lim_{n\to\infty} a_n = a_0$$

$$\underline{|a_1 - a_n| < \frac{1}{n-1}} \rightsquigarrow \lim_{n\to\infty} a_n = a_1$$

FIX m

$$\lim_{n\to\infty} a_n = \underline{a_m}$$

$$0 < a - 1 = (a^{1/n} - 1)(a^{\frac{n-1}{n}} + a^{\frac{n-2}{n}} + \dots + a^{\frac{1}{n}} + 1)$$

$$\left((a^{1/n})^n - 1 \right) \qquad x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \dots + x + 1)$$

$$x = a^{1/n}$$

Assume $a > 1$

$$0 < a^{1/n} - 1 = \frac{a-1}{\sum_{k=0}^{n-1} a^{\frac{k}{n}}}$$

$$\lim_{n \to \infty} \sqrt[n]{a} = 1$$

$$\lim_{n \to \infty} \left(a^{\frac{1}{n}} - 1\right) = 0$$

$$\frac{a-1}{\underbrace{a^{\frac{n-1}{n}}+a^{\frac{n-2}{n}}+\cdots+a^{\frac{1}{n}}+1}_{n\text{ terms}}}$$

$\boxed{a>1}$ If $a^{\frac{1}{n}}<1$

$a^{\frac{1}{n}}\cdot a^{\frac{1}{n}}<1$

$\vdots$

$a=a^{\frac{n}{n}}<1\cdot1\cdot1\cdots1=1$

$$a^{1/n} = \sqrt[n]{a} > 1$$

$$a^{2/n} > 1$$

$$a^{k/n} > 1$$

$$\frac{a-1}{a^{\frac{n-1}{n}} + a^{\frac{n-2}{n}} + \cdots + a^{1/n} + 1} < \frac{a-1}{1+1+1+1+\cdots+1} = \frac{a-1}{n}$$

$$0 < a^{\frac{1}{n}} - 1 < \frac{a-1}{n}$$

$$n a^n \to 0 \qquad \lim_{n\to\infty} n$$

$$a = \frac{1}{1+h}$$

$$n a^n = \frac{n}{(1+h)^n} = \frac{n}{1+nh+\frac{n(n-1)}{2}h^2+\cdots+nh^{n-1}+h^n}$$

$$\lim a_n \cdot b_n = \lim a_n \cdot \lim b_n$$

$$na^n = \cfrac{1}{\overset{0}{\cancel{\frac{1}{n}}} + \underline{n} + \frac{n-1}{2}n^2 + \frac{(n-1)(n-2)}{6}n^3 + \cdots + \underline{n^{n-1}} + \cancel{\frac{n^n}{n}}} \longrightarrow 0$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$f(0) = 0$$

$$f(x+y) = f(x) + f(y)$$

$$f(0) = f(0+0) = f(0) + f(0)$$

$$\Longrightarrow f(0) = 0$$

$$f(2) = f(1) + f(1) = 2f(1)$$

$$f(n) = f(1) + f(n-1) =$$

$$= f(1) + (n-1)f(1)$$

$$= n f(1)$$

$f(-1)$

$1-1=0$

$f(1)+f(-1)=f(0)=0$

$f(-1)=-f(1)$

For any integer k

$$f(k) = kf(1)$$

Let $f(1) = c$.

Let $x \in \mathbb{Q}$

$$x = \frac{p}{q} = p\left(\frac{1}{q}\right)$$

$$f(x) = f\left(\frac{p}{q}\right) = f\left(p \cdot \left(\frac{1}{q}\right)\right)$$

$$= p f\left(\frac{1}{q}\right)$$

$$f\left(\frac{1}{q}\right)$$

$$c = f\left(\frac{q}{q}\right) = q \cdot f\left(\frac{1}{q}\right)$$

$$c = q \cdot f(1/q)$$

$$f(1/q) = \frac{c}{q}$$

$$f(x) = f\left(\frac{p}{q}\right) = pf\left(\frac{1}{q}\right)$$

$$= p \cdot \frac{c}{q} = c\,\frac{p}{q}$$

$$= cx$$

$$a_{n+1} = \frac{a_n - 1}{2}$$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{a_n - 1}{2}$$

$$A = \frac{A-1}{2}$$

$$A = -1$$

$$a_1 = \frac{a_0 - 1}{2} = \frac{a - 1}{2}$$

$$a_2 = \frac{a_1 - 1}{2} = \frac{\frac{a-1}{2} - 1}{2} = \frac{a-3}{4}$$

$$a_3 = \frac{a_2 - 1}{2} = \frac{\frac{a-3}{4} - 1}{2} = \frac{a-7}{8}$$

$$\rightarrow a_n = \frac{a - (2^n - 1)}{2^n}$$

$$a_0 = 1$$

$$a_{n+1} = a_n + \frac{1}{a_n}$$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n + \frac{1}{\lim_{n\to\infty} a_n}$$

$$A = A + \frac{1}{A}$$

$$\frac{1}{A} = 0$$

$$\lim_{x \to a} b^x = b^{\lim_{x \to a} x}$$