

MATH 6101

Fall 2008

Infinite Series and Convergence



Definition

Given any sequence $\{a_n\}$ we associate a new sequence $\{s_n\}$ of *partial sums*:

$$s_n = a_1 + a_2 + a_3 + a_4 + \dots + a_n$$

We define the **series** $\sum a_n$ to be the limit:

$$\sum a_n = \lim_{n \rightarrow \infty} s_n$$

If the sequence of partial sums converges, we say that the infinite series *converges*. Otherwise, we say that the series is *divergent*.

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Examples

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 2 \quad \sum_{n=1}^{\infty} \frac{n^2}{2^n} = 6 \quad \sum_{n=1}^{\infty} \frac{n^3}{2^n} = 26$$

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Examples

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$$

In this case we have seen that:

$$s_n = \sum_{k=0}^n a^k = 1 + a + a^2 + a^3 + \cdots + a^n = \frac{1-a^{n+1}}{1-a}$$

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Examples

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

In this case we have seen that:

$$\begin{aligned} s_n &= \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{(n-1) \cdot n} \\ &= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n}\right) \\ &= 1 - \frac{1}{n} \end{aligned}$$

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Other Examples

Do these converge or diverge?

$$1 + 2 + 3 + 4 + 5 + \cdots + n + \cdots = \sum_{n=1}^{\infty} n$$

$$1 + 1 + 1 + 1 + 1 + \cdots = \sum_{n=1}^{\infty} 1$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n}$$

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{n^2} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

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First Series

$$\sum_{k=1}^{\infty} k = 1 + 2 + 3 + 4 + 5 + \dots + k + \dots$$

$$s_n = 1 + 2 + 3 + 4 + \dots + n$$

$$s_n = \frac{n(n+1)}{2}$$

$$\lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2} = +\infty$$

Thus, the limit of the sequence of partial sums does not exist as a real number, and the series diverges.

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Second Series

$$\sum_{n=1}^{\infty} 1 = 1 + 1 + 1 + 1 + \dots$$

$$s_n = \underbrace{1 + 1 + \dots + 1}_n = n$$

$$\lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} n = +\infty$$

Again, the sequence of partial sums does not exist as a real number, and the series diverges.

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Third Series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} + \dots = \sum_{n=1}^{\infty} \frac{1}{n}$$

This one is more difficult to see, but in 1350 Nicole Oresme proves the following:

$$\begin{aligned} 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots &= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) + \dots \\ &= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) + \dots \\ &> 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots \end{aligned}$$

So this one does not add up to a finite number.

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$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Does this converge or diverge?

We know that $2n^2 \geq n(n+1)$ so

$$\frac{2}{n(n+1)} \geq \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \leq \sum_{n=1}^{\infty} \frac{2}{n(n+1)} = 2$$

Therefore, it does converges.

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Continued

We noted earlier that Euler proved in 1735 that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$$

We also know more:

$$\sum_{n=1}^{\infty} \frac{1}{n^{2k}} = \frac{2^{2k-1} |B_{2k}| \pi^{2k}}{(2k)!}$$

where B_n is the n th *Bernoulli* number. Euler only went through the exponent 26.

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Bernoulli Numbers

The Bernoulli numbers B_n were discovered by Jakob Bernoulli in conjunction with computing the sums of powers:

$$\sum_{k=0}^{m-1} k^n = 0^n + 1^n + 2^n + 3^n + 4^n + \dots + (m-1)^n$$

For example:

$$\sum_{k=0}^n k = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$\sum_{k=0}^n k^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

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Bernoulli Numbers

$$\sum_{k=0}^n k^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$

$$\sum_{k=0}^n k^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$$

$$\sum_{k=0}^n k^5 = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}n^2$$

$$\sum_{k=0}^n k^6 = \frac{1}{7}n^7 + \frac{1}{2}n^6 + \frac{1}{2}n^5 - \frac{1}{6}n^3 + \frac{1}{42}n$$

$$\sum_{k=0}^n k^7 = \frac{1}{8}n^8 + \frac{1}{2}n^7 + \frac{7}{12}n^6 - \frac{7}{24}n^4 + \frac{1}{12}n^2$$

$$\sum_{k=0}^n k^8 = \frac{1}{9}n^9 + \frac{1}{2}n^8 + \frac{2}{3}n^7 - \frac{7}{15}n^5 + \frac{2}{9}n^3 - \frac{1}{30}n$$

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Bernoulli Numbers

$$\sum_{k=0}^n k^9 = \frac{1}{10}n^{10} + \frac{1}{2}n^9 + \frac{3}{4}n^8 - \frac{7}{10}n^6 + \frac{1}{2}n^4 - \frac{1}{12}n^2$$

$$\sum_{k=0}^n k^{10} = \frac{1}{11}n^{11} + \frac{1}{2}n^{10} + \frac{5}{6}n^9 - n^7 + n^5 - \frac{1}{2}n^3 + \frac{5}{66}n$$

$$\sum_{k=0}^n k^{11} = \frac{1}{12}n^{12} + \frac{1}{2}n^{11} + \frac{11}{12}n^{10} - \frac{11}{8}n^8 + \frac{11}{6}n^6 - \frac{11}{8}n^4 + \frac{5}{12}n^2$$

$$\sum_{k=0}^n k^{12} = \frac{1}{13}n^{13} + \frac{1}{2}n^{12} + n^{11} - \frac{11}{6}n^9 + \frac{22}{7}n^7 - \frac{33}{10}n^5 + \frac{5}{3}n^4 - \frac{691}{2730}n$$

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Bernoulli Numbers

Bernoulli then states:

$$\sum_{k=0}^n k^p = \frac{1}{p+1}n^{p+1} + \frac{1}{2}n^p + \frac{p}{2}An^{p-1} + \frac{p(p-1)(p-2)}{2 \cdot 3 \cdot 4}Bn^{p-3}$$

$$+ \frac{p(p-1)(p-2)(p-3)(p-4)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}Cn^{p-5}$$

$$+ \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)(p-6)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}Dn^{p-7} + \dots$$

where

$$A = \frac{1}{6}, \quad B = -\frac{1}{30}, \quad C = \frac{1}{42}, \quad D = -\frac{1}{30}$$

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Bernoulli Numbers

The p th Bernoulli number is the coefficient of x^p in the polynomial describing $\sum k^p$. Other techniques for generating the Bernoulli numbers come from

$$\sum_{k=0}^{n-1} k^p = \frac{1}{p+1} \sum_{k=0}^p \binom{p+1}{k} B_k n^{p+1-k}$$

or

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}$$

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Bernoulli Numbers

n	B_n	n	B_n
0	1	12	-691/2730
1	-1/2	14	7/6
2	1/6	16	-3617/510
4	-1/30	18	43867/798
6	1/42	20	-174611/330
8	-1/30	22	854513/138
10	5/66	24	-236364091/2730

$B_{2n+1} = 0, n > 0$

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Series

We will be able to show later that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges if and only if $p > 1$. The easier proof requires a little calculus.

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Absolute Convergence

If the terms a_n of an infinite series $\sum a_n$ are all nonnegative, then the partial sums $\{s_n\}$ form a non-decreasing sequence.

Therefore, $\sum a_n$ either converges or diverges to ∞ .

$\sum |a_n|$ is non-decreasing for any sequence.

The series $\sum a_n$ is said to *converge absolutely* if $\sum |a_n|$ converges.

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Conditional Convergence

A series *converges conditionally*, if it converges, but not absolutely.

- The series $\sum (-1)^n$ diverges.
- The series $\sum (1/2)^n$ converges absolutely.
- The series $\sum (-1)^{n+1}/n$ converges conditionally (this series is called alternating harmonic series).

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Order of Summation

Theorem:

(i) Let $\sum a_n$ be an absolutely convergent series. Then any rearrangement of terms in that series results in a new series that is also absolutely convergent to the same limit.

(ii) Let $\sum a_n$ be a conditionally convergent series. Then, for any real number c there is a rearrangement of the series such that the new resulting series will converge to c .

(To be proven later)

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Algebra of Series

Let $\sum a_n$ and $\sum b_n$ be two absolutely convergent series.
Then

- (i) The sum of the two series is again absolutely convergent. $\sum(a_n + b_n) = \sum a_n + \sum b_n$
- (ii) The difference of the two series is again absolutely convergent. $\sum(a_n - b_n) = \sum a_n - \sum b_n$
- (iii) The product of the two series is again absolutely convergent. Its limit is the product of the limit of the two series.

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Algebra of Series

The Cauchy product of two series $\sum a_n$ and $\sum b_n$ is defined as follows. The Cauchy product is

$$\left(\sum_{n=m}^{\infty} a_n \right) \left(\sum_{n=m}^{\infty} b_n \right) = \left(\sum_{n=m}^{\infty} c_n \right) \text{ where } c_n = \sum_{k=0}^n a_k b_{n-k}$$

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nth Term Test

Theorem: If $\sum a_n$ converges then $\{a_n\} \rightarrow 0$.

Metaproof: If $\sum a_n$ converges, then the sequence of partial sums converges $\{s_n\} \rightarrow L$. Note that the sequence $\{s_{n-1}\}$ also converges to L . Thus,

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (s_n - s_{n-1}) = \lim_{n \rightarrow \infty} s_n - \lim_{n \rightarrow \infty} s_{n-1} = L - L = 0.$$

Corollary: If $|a| \geq 1$ then $\sum a^n$ diverges.

nth Term Test: If $\lim a_n \neq 0$, then $\sum a_n$ diverges.

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Comparison Test

Theorem: If $\sum a_n$ and $\sum b_n$ are series so that $0 \leq a_n \leq b_n$.

Then

if $\sum b_n$ converges so does $\sum a_n$;

if $\sum a_n$ diverges so does $\sum b_n$.

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Comparison Test

Proof: Set

$$d_n = a_0 + a_1 + a_2 + \dots + a_n$$

$$e_n = b_0 + b_1 + b_2 + \dots + b_n$$

$\{d_n\}$ and $\{e_n\}$ are increasing sequences.

$$0 \leq d_n \leq e_n$$

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Comparison Test

Each converges or diverges depending on whether it is bounded or not.

$\sum b_n$ converges $\implies \{e_n\}$ converges $\implies \{e_n\}$ bounded $\implies \{d_n\}$ bounded $\implies \{d_n\}$ converges $\implies \sum a_n$ converges

$\sum a_n$ diverges $\implies \{d_n\}$ diverges $\implies \{d_n\}$ unbounded $\implies \{e_n\}$ unbounded $\implies \{e_n\}$ diverges $\implies \sum b_n$ diverges

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Examples

$\sum_{n=1}^{\infty} \frac{1}{n2^n}$ (converges) or diverges?

$\frac{1}{n2^n} < \frac{1}{2^n}$ and we know that the latter converges

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Limit Comparison Test

Theorem:

Let $\sum a_n$ and $\sum b_n$ be two series. Suppose also

$r = \lim |a_n/b_n|$ exists and $0 < r < +\infty$.

Then $\sum a_n$ converges absolutely if and only if $\sum b_n$ converges absolutely.

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Limit Comparison Test

Proof: $r = \lim |a_n/b_n|$ and r is a positive real number. There are constants c and C ,

$$0 < c < C < +\infty$$

so that for some $N > 1$ if $n > N$

$$c < |a_n/b_n| < C.$$

Assume $\sum a_n$ converges absolutely. For $n > N$, $c|b_n| < |a_n|$. Therefore, $\sum b_n$ converges absolutely by the Comparison Test.

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Limit Comparison Test

Assume that $\sum b_n$ converges absolutely.

For $n > N$, $|a_n| < C|b_n|$.

$C \sum b_n$ converges absolutely.

$\sum a_n$ converges absolutely by Comparison Test.

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Cauchy Condensation Test

Theorem:

Suppose $\{a_n\}$ is a decreasing sequence of positive terms. Then the series $\sum a_n$ converges if and only if the series $\sum 2^k a_{2^k}$ converges.

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p -series Test

Corollary:

For a positive number p , $\sum 1/n^p$ converges if and only if $p > 1$.

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p -series Test

Proof:

If $p < 0$ then the sequence $\{1/n^p\}$ diverges to infinity. Hence, the series diverges by the n th Term Test.

If $p > 0$ then consider the series

$$\sum 2^n a_{2^n} = \sum 2^n / (2^n)^p = \sum (2^{1-p})^n.$$

By the geometric series,

- if $0 < p \leq 1$, $2^{1-p} \geq 1$, so right-hand series diverges;
- if $p > 1$ then $2^{1-p} < 1$, so right-hand series converges.

Now the result follows from the Cauchy Condensation Test.

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Root Test

Theorem:

Let $\sum a_n$ be a series and let

$$\alpha = \limsup |a_n|^{1/n}.$$

The series $\sum a_n$

- i. converges absolutely if $\alpha < 1$,
- ii. diverges if $\alpha > 1$.
- iii. Otherwise $\alpha = 1$ and the test gives no information.

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Root Test

Proof:

- i) Suppose that $\alpha < 1$. Then choose an $\varepsilon > 0$ so that $\alpha + \varepsilon < 1$. By definition of \limsup there exists N so that $\alpha - \varepsilon < \text{lub}\{|a_n|^{1/n} \mid n > N, \alpha + \varepsilon\}$. In particular, $|a_n|^{1/n} < \alpha + \varepsilon$ for $n > N$, so $|a_n| < (\alpha + \varepsilon)^n$ for $n > N$. Since $0 < \alpha + \varepsilon < 1$, the geometric series $\sum(\alpha + \varepsilon)^n$ converges. By the Comparison Test $\sum|a_n|$ converges. This means that $\sum a_n$ also converges.

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Root Test

Proof:

- i) If $\alpha > 1$, then there is a subsequence of $|a_n|^{1/n}$ that has limit $\alpha > 1$. That means that $|a_n| > 1$ for infinitely many n . The sequence $\{a_n\}$ cannot converge to 0, so $\sum a_n$ cannot converge.
- ii) For the series $\sum 1/n$ and $\sum 1/n^2$, $\alpha = 1$. The harmonic series diverges and the other converges, so $\alpha = 1$ can not guarantee either convergence or divergence of the series.

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Ratio Test

Theorem:

The series $\sum a_n$

- i. converges absolutely if $\limsup |a_{n+1}/a_n| < 1$,
- ii. diverges if $\liminf |a_{n+1}/a_n| > 1$.
- iii. Otherwise
 $\liminf |a_{n+1}/a_n| \leq 1 \leq \limsup |a_{n+1}/a_n|$
 and the test gives no information.

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Alternating Series Test

Theorem:

If $a_1 \geq a_2 \geq \dots \geq a_n \geq \dots \geq 0$ and $\{a_n\}$ converges to zero, then the alternating series $\sum(-1)^n a_n$ converges.

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Problems

$$\lim_{n \rightarrow \infty} \sqrt[n]{a^n + b^n} = \max\{a, b\} \text{ for } a \geq 0 \text{ and } b \geq 0.$$

$$\lim_{n \rightarrow \infty} (n - \sqrt{n+a} \sqrt{n+b}) = -\frac{a+b}{2}$$

$$\lim_{n \rightarrow \infty} \frac{a^n - b^n}{a^n + b^n} = \begin{cases} 0 & \text{if } a = b \neq 0 \\ 1 & \text{if } |a| > |b| \\ -1 & \text{if } |a| < |b| \\ \text{undefined} & \text{if } a = b = 0 \end{cases}$$

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Problems

$$\lim_{n \rightarrow \infty} \frac{2^{n^2}}{n!}$$

$$2n^2 \geq n(n+1) \Rightarrow n^2 \geq \frac{n(n+1)}{2} = n + (n-1) + \dots + 2 + 1$$

$$2^{n^2} \geq 2^{\frac{n(n+1)}{2}} = 2^n \times 2^{n-1} \times \dots \times 2^2 \times 2$$

$$\frac{2^{n^2}}{n!} \geq \frac{2^n \times 2^{n-1} \times \dots \times 2^2 \times 2}{n \times (n-1) \times \dots \times 2 \times 1} \rightarrow \infty$$

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Problems

$$\sum_{n=1}^{\infty} \frac{\sin(n\theta)}{n^2}$$

Since $\sin n\theta \leq 1$ for all n we have that

$$\frac{\sin(n\theta)}{n^2} \leq \frac{1}{n^2}$$

so this series converges by the Comparison Test with the series .

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

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Problems

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n^2 - 1}}$$

This diverges by the Comparison Test with the series

$$\sum_{n=2}^{\infty} \frac{1}{n^{2/3}}$$

which diverges by the p -series Test.

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Problems

$$\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n}$$

Since $\lim (\log n)/n = 0$ and $a_{n+1} \leq a_n$, by the Alternating Series Test this series converges.

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Problems

$$\sum_{n=1}^{\infty} \frac{n^2}{n!} \quad \text{Use the Ratio Test}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)^2 n!}{(n+1)! n^2} = \lim_{n \rightarrow \infty} \frac{n+1}{n^2} = 0$$

This series converges.

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Problems

$$\sum_{n=1}^{\infty} \frac{\log n}{n}$$

$$\frac{\log n}{n} > \frac{1}{n} \quad \text{So the series diverges by the Comparison Test.}$$

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Problems

$$\sum_{n=2}^{\infty} \frac{1}{\log n}$$

$$\log n < n \Rightarrow \frac{1}{n} < \frac{1}{\log n}$$

So the series diverges by the Comparison Test.

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Problems

$$\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$$

Use the Root Test.

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \frac{1}{\log n} = 0 < 1$$

So the series converges by the Root Test.

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Problems

$$\sum_{n=0}^{\infty} \frac{n^2}{n^3 + 1}$$

This series diverges by the Limit Comparison Test with $\sum 1/n$.

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Problems

$$\sum_{n=2}^{\infty} \frac{1}{n^2 \log n}$$

$$\frac{1}{n^2 \log n} < \frac{1}{n^2}$$

So the series converges by the Comparison Test.

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Problems

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$$

Use the Limit Comparison Test with $\sum 1/n$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{n^{1+1/n}}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n}{n^{1+1/n}} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot n^{1/n}} = \lim_{n \rightarrow \infty} \frac{1}{n^{1/n}} = 1 < +\infty$$

Thus, the series diverges.

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