

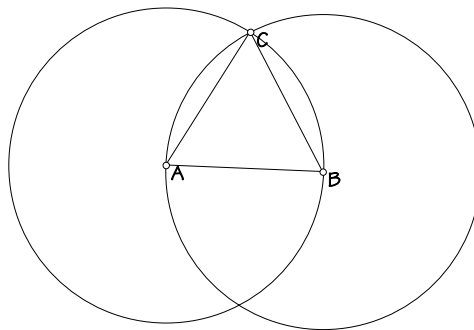
Worksheet 1

Equilateral Triangles

The first construction in Euclid's *Elements* is the equilateral triangle. Countless other constructions in the *Elements* depend on being able to construct an equilateral triangle with compass and straightedge. In this worksheet we will construct an equilateral triangle on a given segment, and then create a script out of this construction. We will do this so that we do not have to repeat the entire construction each time we need an equilateral triangle, but only need to use the script.

Sketch

1. Construct \overline{AB} .
2. Construct circle AB , making sure that you use point A for the center and point B for the radius-defining point.
3. Construct circle BA .
4. Construct \overline{AC} and \overline{CB} , where C is a point of intersection of the two circles.
5. Hide the circles.
6. Measure the three angles.
7. Select the triangle and from the **Work** item choose **Make Script**.

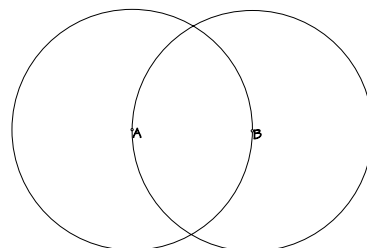


Worksheet 2

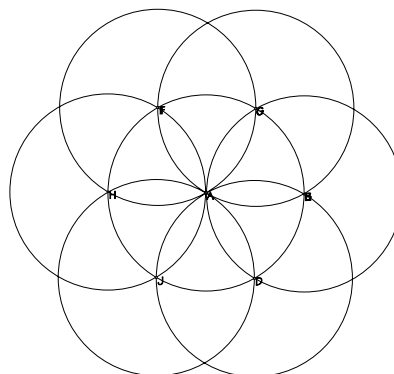
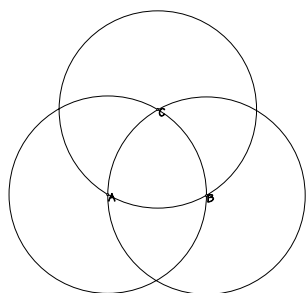
Daisy Designs

A daisy design is a simple design that you can create using only a compass. From the basic daisy, you can create more complex designs based on the regular hexagon.

1. Construct circle AB .
2. Construct circle BA .
3. Drag point A and point B to confirm that both circles are controlled by these two points.
4. Construct point C and D , the two points of intersection of these circles.

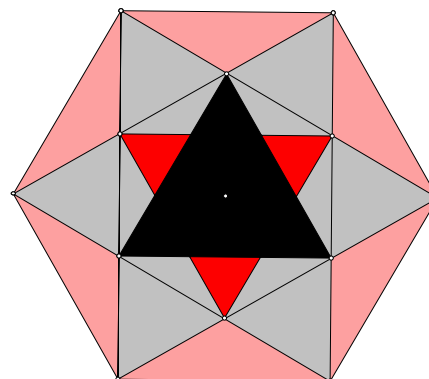
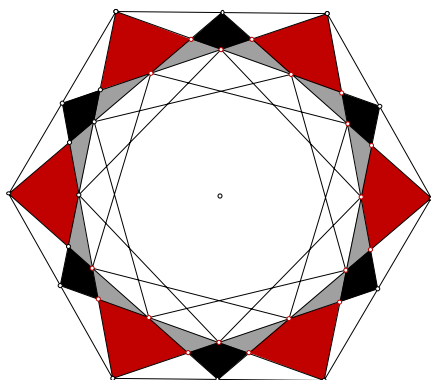


5. Construct a circle from point C to point A . Do **not** construct a circle from point D yet. Save this for last.
6. Continue constructing circles from intersections to point A . All these circles should have equal radii. The last circle you construct should be centered at point D . When you are done, your sketch should look like the figure below right. You should be able to drag it without making it fall apart.



7. Use the **Segment** tool to add some line to your design; then drag point B and observe the way your design changes.

The six points of your daisy define six vertices of a regular hexagon. You can use these points as the basis for hexagon or star designs like the ones below.



Worksheet 3

Copying a Line Segment

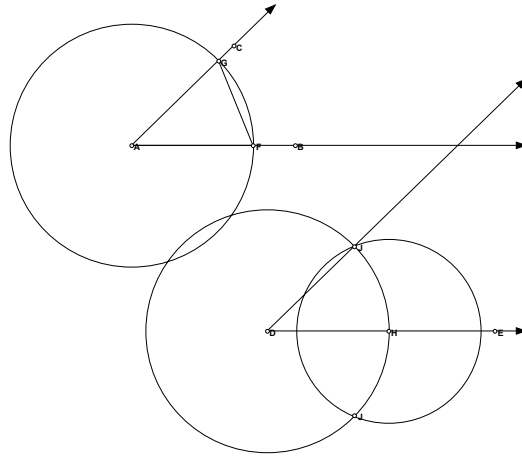
This method for duplication a line segment with *Sketchpad* is equivalent to the standard compass-and-straightedge construction. However, *Sketchpad's* compass (the **Circle** tool) is collapsible, like Euclid's compass, meaning that after you make a circle you can't just pick up the tool, move it somewhere else, and make another circle the same size. As soon as you pick Euclid's compass up (or release the mouse button when you are using *Sketchpad's* **Circle** tool), you lose your compass setting. Duplicating a segment using only *Sketchpad's* freehand tools (or Euclid's tools) is more complicated than you would expect. We avoid this problem using the **Construct** menu.

1. Construct \overline{AB} . This is your line segment.
2. Construct a point C . This is one endpoint of your new segment.
3. Select \overline{AB} and point C ; then, in the **Construct** menu, choose **Circle By Center+Radius**.
4. Construct \overline{CD} , where point D is on the circle.
5. Hide the circle.
6. Measure AB and CD .

Worksheet 4

Copying an Angle

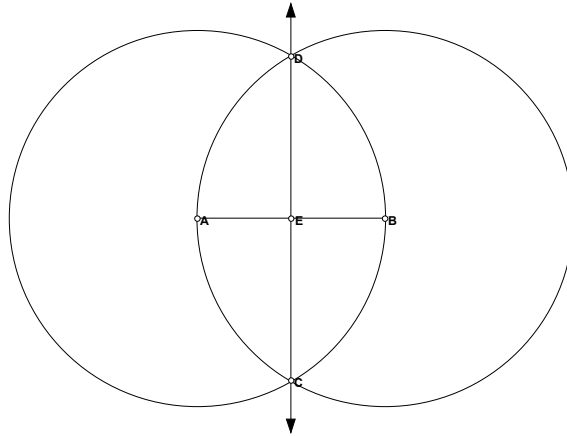
We are going to duplicate an angle as we do with a compass and straightedge. You can save this construction as a script if you want.



Worksheet 5

Perpendicular Bisectors

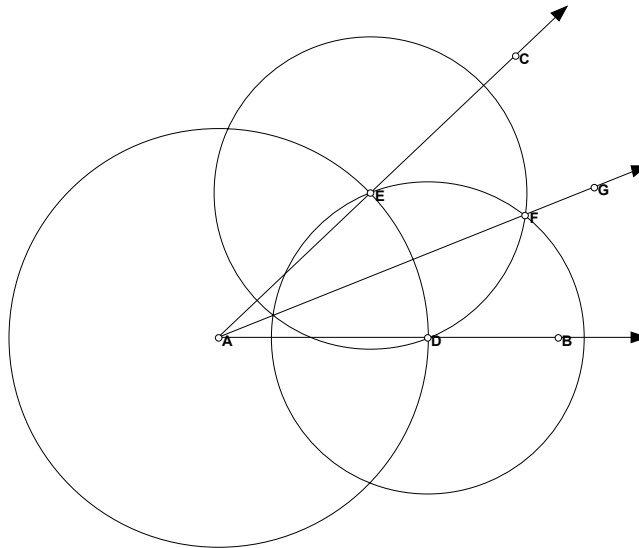
We are going to construct the perpendicular bisector of a segment using the Euclidean construction.



Worksheet 6

Angle Bisectors

We can bisect any angle by choosing Angle bisector. However, we need to see how to do it using our Euclidean construction tools.



Worksheet 7

Trisecting an Angle

Trisecting an angle with a compass and straightedge is a construction problem that has occupied professional and amateur mathematicians for centuries. Even though it has been proven that the construction cannot be done, countless people still think they have found a solution. Others enjoy slightly bending the rules that govern compass-and-straightedge constructions to devise simple angle trisection methods. With Sketchpad, we can measure the angle and then rotate an angle through the marked measurement.

1. In Preferences, set Angle Unit to directed degrees.
2. Construct rays \overrightarrow{AB} and \overrightarrow{AC} for form an angle you will trisect.
3. Measure $\angle BAC$.
4. Calculate $\frac{\angle BAC^\circ}{3}$.
5. Select the calculation; then, in the Transform menu, choose Mark Angle Measurement.
6. Mark point A as a center of rotation.
7. Rotate \overrightarrow{AB} by the marked angle.
8. Rotate this new ray again by the marked angle.
9. Draw a triangle with extended sides use the Line tool. Trisect the three interior angles. Find an interesting shape formed by the intersecting trisectors.

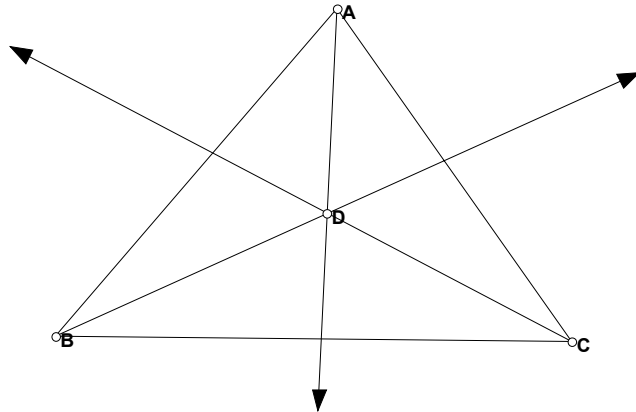
Worksheet 8

Incenter of a Triangle

Use the Angle bisector tool for this construction.

Since the distances from the incenter to each of the sides are the same, we should be able to inscribe a circle in the triangle. Do so. Move the triangle around. Does the circle stay inscribed? Is the incenter always in the interior of the triangle?

1. Construct a triangle.
2. Construct the angle bisectors of each of the three angles.
3. Mark the point of intersection of the angle bisectors. This is the incenter D.
4. Hide the angle bisectors. Select the triangle and the incenter and make a script. Save the script as INCENTER.
5. Measure the angles and the bisectors.
6. Find the points at which the angle bisectors intersect the opposite sides. Find the distance from each vertex to D and from D to each of the points of intersection found above. What is the ratio of the distance from the vertex to D and from D to the point on the opposite side.



Worksheet 9

Circumcenter of a Triangle

Use the Perpendicular bisector tool for this construction.

Sketch

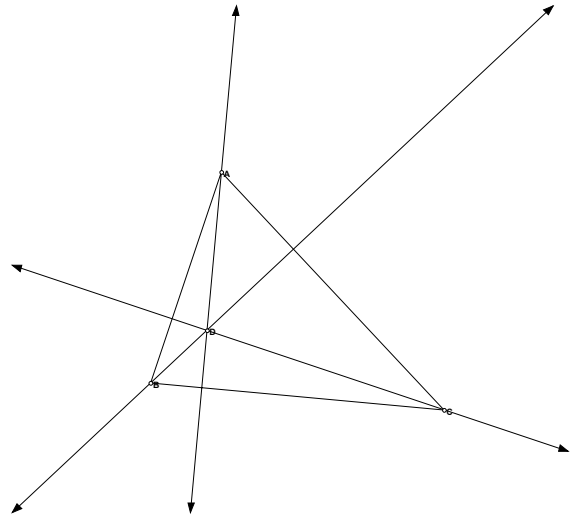
1. Construct a triangle.
2. Construct the perpendicular bisector of each side.
3. Find the point of intersection of these three rays. Hide the rays and create a script to create the circumcenter of a triangle.
4. Choose the circumcenter and one of the vertices of the triangle. Construct a circle with center the circumcenter and passing through the vertex. What other points does this circle contain? Where is the triangle in relation to this circle?

Worksheet 10

Orthocenter of a Triangle

Use the Perpendicular line tool for this construction.

1. Construct a line through each vertex perpendicular to the opposite side of the triangle.
2. Construct the point of intersection of these three perpendiculars. This is called the orthocenter of the triangle.
3. Make a script to construct the orthocenter of a triangle.



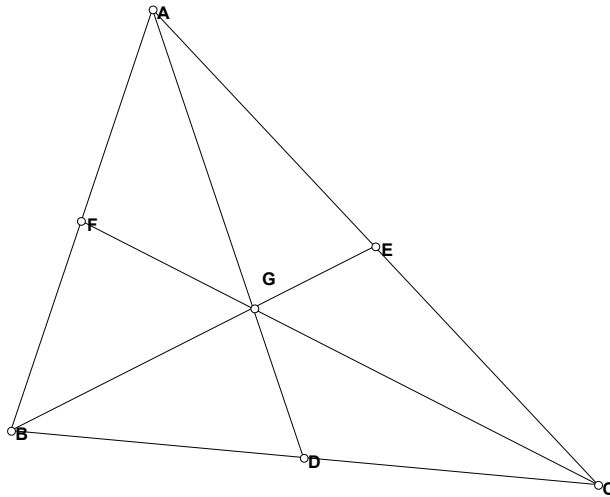
Worksheet 11

Centroid of a Triangle

Use the Midpoint tool for this construction.

Does the third median pass through the point of intersection of the other two medians? Drag any vertex or side of your triangle. Do the three medians always intersect in a single point? Into what ratio does the centroid divide each median?

Measure the areas of the six smaller triangles? Is there anything remarkable about these numbers? Does it stay the same as the triangle changes?



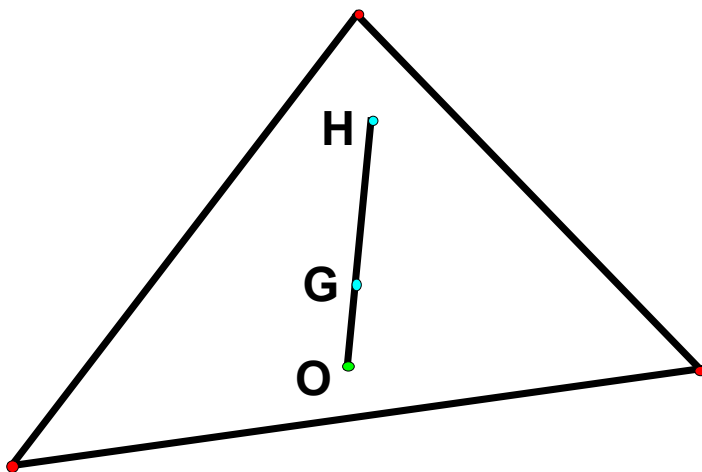
Worksheet 12

The Euler Segment

The various points of concurrency of a triangle make interesting investigations: the properties of the centroid, the circumcenter, the orthocenter, and the incenter are quite remarkable. In this activity you will investigate the relationship of three of these points.

1. Construct a triangle.
2. Construct the centroid of this triangle and label the centroid, G. Hide the medians.
3. Construct the circumcenter of this triangle and label it O.
4. Construct the orthocenter of this triangle and label it H.
5. Construct the segment from the circumcenter to the orthocenter. (This is the *Euler segment*.) Drag any vertex and note what happens.
6. Measure the distance from the circumcenter to the centroid, and the distance from the orthocenter to the centroid.

When you drag a vertex to change the triangle's shape, observe the behavior of the centroid, circumcenter and orthocenter. Also observe the distance measurements. Calculate the ratio of the measurements and drag again.



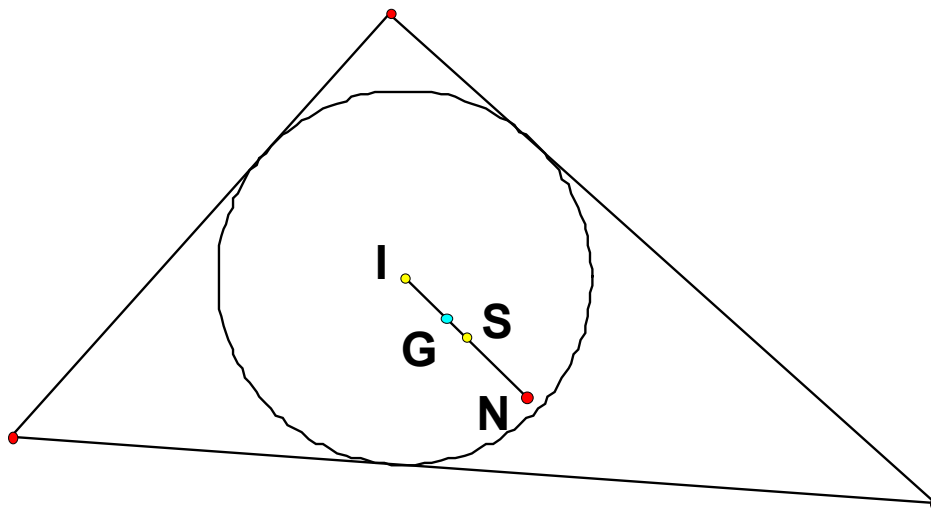
Worksheet 13

The Nagel Segment

We want to look at the Nagel segment and the relationships between the Spieker circle and the incircle of a triangle.

1. Construct a triangle.
2. Construct the centroid of this triangle and label the centroid, G . Hide the medians.
3. Construct the incenter, I , and the incircle of the triangle.
4. The lines connecting the point of tangency of each excircle of $\triangle ABC$ to the opposite vertex will intersect in a point, called the Nagel point, N . We need to construct the excircles.
5. One more point of interest is the center of the incircle for $\triangle ABC$'s medial triangle. This circle is called the Spieker circle and its center is called the Spieker point, S .
6. The Nagel segment is a line segment from the incenter, I , to the Nagel point, N , which contains the Spieker point, S , and the centroid, G .
7. Draw the triangle whose vertices are the midpoints of the segments that join the vertices of $\triangle ABC$ with its Nagel point. What is the incircle for this triangle?
8. Measure the distance from the incenter to the centroid, and the distance from the Nagel point to the centroid. What is the ratio of these two distances?
9. Measure the distance from the incenter to the Spieker point. What is the ratio of this distance to the length of the Nagel segment?
10. Measure the radius of the incircle. Measure the radius of the Spieker circle. What is the ratio of the radius of the incircle to the radius of the Spieker?

Observe the distance measurements as you drag a vertex. Do the ratios remain the same?



Worksheet 14

The Surfer and the Spotter

Two shipwrecked survivors manage to swim to a desert island. As it happens, the island is in the shape of a perfect equilateral triangle. The survivors turn out to have very different dispositions. Sarah soon discovers that the surfing is outstanding on all three of the island's coasts. She crafts a surfboard from a fallen tree and surfs every day. Spencer, on the other hand, is more a social animal and sorely misses civilization. Every day he goes to a different corner of the island and searches the waters for passing ships. Each castaway wants to locate a home in the place that best suits his/her respective needs. (They have no interest in living in the same place, though if it turns out to be advantageous, neither is against the idea either.) Sarah wants to find the spot closest to her three beaches. (She visits them with equal frequency.) Spencer wants his house to be situated so that he can beat the shortest possible paths to the three corners of the island. In other words, the sum of the distances to the sides of the triangle must be minimized for Sarah; the sum of the distances to the vertices must be minimized for Spencer. Where should they locate their huts?

Sketch

Step 1: Construct an equilateral triangle ABC .

Step 2: Construct DA , DB , and DC , where D is any point inside the triangle.

Step 3: If you want, re-label point D as Spencer.

Step 4: Construct E anywhere inside the triangle.

Step 5: Construct a perpendicular to AB through E . Repeat the construction to BC and AC through E .

Step 6: Construct EF , EG , EH , where F , G , and H are the points where the perpendiculars intersect the sides of the triangle.

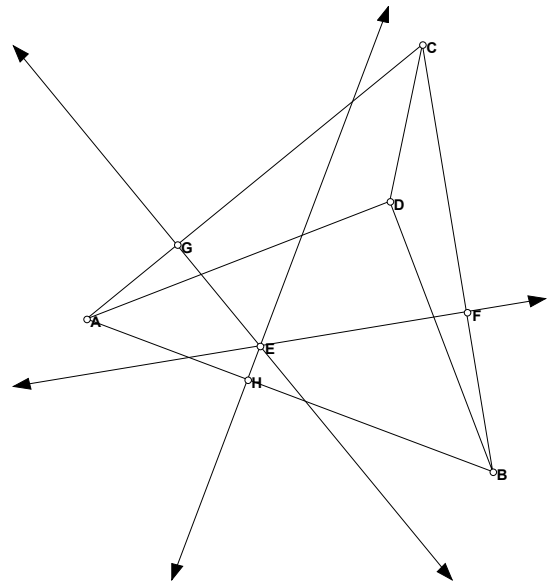
Step 7: Hide the perpendicular lines. Re-label E as Sarah if you wish.

Step 8: Measure DA , DB , and DC .

Step 9: Calculate $DA + DB + DC$.

Step 10: Measure EF , EG , and EH .

Step 11: Calculate $EF + EG + EH$



Move the points D and E around inside your triangle. See if you can find the best locations for each castaway. What are these locations?

Worksheet 15

A Gothic Window

Make a **precise** construction of the pictured gothic church window.

The arch is formed with two intersecting congruent circles: one with radius AB and the other with radius BA .

The circle is inscribed in this arch.

The eight small arches are formed from tangent circles inscribed in the larger circle.

The proportions of the rectangular part are up to you. You may add additional designs as you see fit.

You should include the following three things in the project that you turn in.

1. Your calculations where you found r in terms of a .
2. A preliminary picture of your window where you have not hidden all the extra things that you used in your construction (this should show your method of construction).
3. The final picture of the window.

