MATH 6118-090
Non-Euclidean Geometry

Exercise Set #4

1. Suppose x is a root of

$$x^3 - 5x^2 + 7x - 2 = 0 .$$

Is the length x constructible? Explain your answer.

2. Recall that the *circumcenter* of a triangle is the point at which the perpendicular bisectors of the sides meet.
   a. Is it possible for the circumcenter to lie on a vertex of the triangle? If so, under what conditions? If not, why not?
   b. Is it possible for the circumcenter to line on a side of the triangle? If so, under what conditions? If not, why not?
   c. If the triangle is equilateral, where is the circumcenter?

3. Recall that the *incenter* of a triangle is the point at which the angle bisectors meet.
   a. Is it possible for the incenter to lie on a vertex of the triangle? If so, under what conditions? If not, why not?
   b. Is it possible for the incenter to line on a side of the triangle? If so, under what conditions? If not, why not?
   c. If the triangle is equilateral, where is the incenter?

4. Recall that the *orthocenter* of a triangle is the point at which the altitudes meet.
   a. Is it possible for the orthocenter to lie on a vertex of the triangle? If so, under what conditions? If not, why not?
   b. Is it possible for the orthocenter to line on a side of the triangle? If so, under what conditions? If not, why not?
   c. If the triangle is equilateral, where is the orthocenter?

5. Recall that the *centroid* of a triangle is the point at which the medians meet.
   a. Is it possible for the centroid to lie on a vertex of the triangle? If so, under what conditions? If not, why not?
   b. Is it possible for the centroid to line on a side of the triangle? If so, under what conditions? If not, why not?
   c. If the triangle is equilateral, where is the centroid?

6. (See pg 39, #2) The first known solution to the problem of trisecting an angle is attributed to Hippocrates. Draw a perpendicular from point C on one side of the given angle $\angle CAB$ to a point D on the other side of the angle. Then construct a rectangle $\square CDAF$. Draw the ray $\overrightarrow{FC}$ and locate a point E on $\overrightarrow{FC}$ such that $HE = 2AC$. Hippocrates claims now that $\angle EAB = \frac{1}{3}\angle CAB$. Why is this true? How does this violate the "straightedge and ruler" constraint on the constructions?