MATH 6102 — SPRING 2009
ASSIGNMENT 1

January 26, 2009
Due February 2, 2009

Evaluate the following limits.

1. $\lim_{x\to 2}(x^2 - 4x)$
2. $\lim_{x\to 0}\dfrac{3^x - 3^{-x}}{3^x + 3^{-x}}$
3. $\lim_{x\to 2}\dfrac{x^2 - 4}{x^2 - 5x + 6}$
4. $\lim_{x\to -1}\dfrac{x^2 + 3x + 2}{x^2 + 4x + 3}$
5. $\lim_{x\to 2}\dfrac{x - 2}{\sqrt{x^2 - 4}}$
6. $\lim_{x\to \infty}\dfrac{7x^9 - 4x^4 + 2x - 13}{-3x^9 + x^8 - 5x^2 + 2x}$
7. $\lim_{x\to \infty}\dfrac{3^x - 3^{-x}}{3^x + 3^{-x}}$
8. Show that $\lim_{x\to\infty} x - \sqrt{x^2 - 1} = 0$.
9. Show that the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ gets arbitrarily close to the asymptote $y = \dfrac{b}{a}x$ as x approaches ∞.
10. Is it possible that $\lim_{x\to a}[f(x) + g(x)]$ might exist even though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists? If it is true, then find an example of two such functions.
11. Is it possible that $\lim_{x\to a}[f(x)g(x)]$ might exist even though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists? If it is true, then find an example of two such functions.

12. In the figure below, the circle C is fixed and has equation $(x-1)^2+y^2=1$. The circle G is centered at the origin, has radius r, and is shrinking to the origin. The point P is the point $(0,r)$, Q is the upper point of intersection of the two circles and R is the point where the line PQ intersects the x-axis. What happens to the point R as the circle G shrinks, that is, what is $\lim_{r\to 0} R$?