ASSIGNMENT 3

16-February-2009

1. Express all the hyperbolic functions in terms of $\sinh x$. Given $\cosh x = 2$ find the values of the other functions.

2. (a) Show that

$$(\cosh u_1 + \sinh u_1)(\cosh u_2 + \sinh u_2) = \cosh(u_1 + u_2) + \sinh(u_1 + u_2).$$

   (b) Show that for any positive integer $n > 0$

$$\prod_{i=1}^{n}(\cosh u_i + \sinh u_i) = \cosh\left(\sum_{i=1}^{n} u_i\right) + \sinh\left(\sum_{i=1}^{n} u_i\right).$$

   (c) What does this become if $u_1 = u_2 = \cdots = u_n = u$?

3. Evaluate the following integral in terms of hyperbolic trigonometric functions

$$\int \frac{1}{\sqrt{4 + x^2}}\, dx$$

4. Differentiate the following functions.

   (a) $f(x) = 3x \tanh(4x)$.

   (b) $g(x) = 5x \operatorname{sech}(4x) - 21 \tanh^3(4x)$.

5. (a) Use the substitution $x = \cosh u$, $u > 0$ to show that

$$\int \frac{1}{\sqrt{x^2 - 1}}\, dx = \cosh^{-1}(x) + C$$

   for $x > 1$.

   (b) Use the substitution $x = \sec u$, $0 < u < \frac{\pi}{2}$, to show that

$$\int \frac{1}{\sqrt{x^2 - 1}}\, dx = \ln\left|x + \sqrt{x^2 - 1}\right| + C$$

   for $x > 1$.

   (c) Use the above to show that

$$\cosh^{-1}(x) = \ln\left|x + \sqrt{x^2 - 1}\right|$$

   for $x > 1$.

6. We define the function $\operatorname{glog}(x) = y$ if and only if $x = \dfrac{e^y}{y}$. Find the derivative of $\operatorname{glog}(x)$.

7. Find the second derivative of the Lambert W function.

8. Using the Lambert W function, solve $x \log_b x = a$ for x.