ASSIGNMENT 4

16-March-2009

1. The number a is called a **double root** of the polynomial function f if $f(x) = (x-a)^2 g(x)$ for some polynomial function g. Prove that a is a double root of f if and only if a is a root of f and f'.

2. Prove that it is impossible to write $x = f(x)g(x)$ where f and g are differentiable and $f(0) = g(0) = 0$.

3. Suppose that f is differentiable at a. Prove that

$$f'(a) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}.$$

4. Find the first derivative of each of the following:

   (a) $\sin(\cos(x))$

   (b) $\sin\left(\dfrac{\cos x}{x}\right)$

   (c) $\dfrac{\sin(\cos x)}{x}$

5. Find $f(f'(x))$ if $f(x) = \dfrac{1}{x}$.

6. Let $f(x)$ be differentiable on $(0, \infty)$ and assume that $\lim\limits_{x \to \infty} [f(x) + f'(x)] = L$.

   (a) Find $\lim\limits_{x \to \infty} f(x)$.

   HINT: Apply l'Hospital's Rule to $f(x) = \dfrac{e^x f(x)}{e^x}$.

   (b) If L is a number, find $\lim\limits_{x \to \infty} f'(x)$.

   HINT: Observe that $f'(x) = [f(x) + f'(x)] - f(x)$.

7. If f is differentiable on $(0, \infty)$ and $\lim\limits_{x \to \infty} f'(x) = L$, and $\lim\limits_{n \to \infty} f(n)$ exists as a number, then what must be the value of L?
   HINT: Apply l'Hospital's Rule to $\dfrac{f(x)}{x}$.