Hyperbolic Functions

Hyperbolic cosine of x: $\cosh x = \dfrac{e^x + e^{-x}}{2}$

Hyperbolic sine of x: $\sinh x = \dfrac{e^x - e^{-x}}{2}$

Hyperbolic tangent: $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Hyperbolic cotangent: $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Hyperbolic secant: $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

Hyperbolic cosecant: $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

Identities

$\sinh x + \cosh x = e^x$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh^2 x - \sinh^2 x = 1$

$\tanh^2 x = 1 - \operatorname{sech}^2 x$

$\coth^2 x = 1 + \operatorname{csch}^2 x$

$\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

$\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

Derivatives

$\dfrac{d}{dx}(\sinh u) = \cosh u \dfrac{du}{dx}$

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$\dfrac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \dfrac{du}{dx}$

Integrals

$\int \sinh u \, du = \cosh u + C$

$\int \cosh u \, du = \sinh u + C$

$\int \operatorname{sech}^2 u \, du = \tanh u + C$

$\int \operatorname{csch}^2 u \, du = -\coth u + C$

$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$

$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$

Useful Identities

$\operatorname{sech}^{-1} x = \cosh^{-1}\dfrac{1}{x}$ $\quad$ $\operatorname{csch}^{-1} x = \sinh^{-1}\dfrac{1}{x}$ $\quad$ $\coth^{-1} x = \tanh^{-1}\dfrac{1}{x}$

Derivatives of Inverse Hyperbolic Functions

$\dfrac{d\left(\sinh^{-1} u\right)}{dx} = \dfrac{1}{\sqrt{1+u^2}}\dfrac{du}{dx}$

$\dfrac{d\left(\cosh^{-1} u\right)}{dx} = \dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}, \quad u > 1$

$\dfrac{d\left(\tanh^{-1} u\right)}{dx} = \dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad |u| < 1$

$\dfrac{d\left(\coth^{-1} u\right)}{dx} = \dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad |u| > 1$

$\dfrac{d\left(\operatorname{sech}^{-1} u\right)}{dx} = \dfrac{-1}{u\sqrt{1-u^2}}\dfrac{du}{dx}, \quad 0 < u < 1$

$\dfrac{d\left(\operatorname{csch}^{-1} u\right)}{dx} = \dfrac{-1}{u\sqrt{1+u^2}}\dfrac{du}{dx}, \quad u \neq 0$

Logarithm Formulas for Evaluating Inverse Hyperbolic Functions

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \quad -\infty < x < \infty$

$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right), \quad x \geq 1$

$\tanh^{-1} x = \dfrac{1}{2}\ln\dfrac{1+x}{1-x}, \quad |x| < 1$

$\operatorname{sech}^{-1} x = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right), \quad 0 < x \leq 1$

$\operatorname{csch}^{-1} x = \ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1+x^2}}{|x|}\right), \quad x \neq 0$

$\coth^{-1} x = \dfrac{1}{2}\ln\dfrac{x+1}{x-1}, \quad |x| > 1$

Integrals of Inverse Hyperbolic Functions

$\int \dfrac{du}{\sqrt{1+u^2}} = \sinh^{-1} u + C$

$\int \dfrac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + C, \quad u > 1$

$\int \dfrac{du}{1-u^2} = \begin{cases} \tanh^{-1} u + C & \text{if } |u| < 1 \\ \coth^{-1} u + C & \text{if } |u| > 1 \end{cases}$

$\int \dfrac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1}|u| + C = -\cosh^{-1}\left(\dfrac{1}{|u|}\right) + C$

$\int \dfrac{du}{u\sqrt{1+u^2}} = -\operatorname{csch}^{-1}|u| + C = -\sinh^{-1}\left(\dfrac{1}{|u|}\right) + C$