An innovative approach to the use of games as teaching tools for mathematics

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Mathematically rich games can make mathematics real for students. Because students of all ages enjoy games, they are motivated to learn strategies to succeed in the game. In this paper, the authors will, first, explore the mathematics education literature to explain why mathematically rich games are useful in the classroom. Then, they will describe one of the games that they presented to elementary school students in central North Carolina. Finally, they will present evidence that these games have been useful in the mathematics classroom.

General Mathematics Classroom

In 1989 and 1991, the National Council of Teachers of Mathematics published two sets of recommendations regarding the teaching and learning of mathematics (NCTM, 1989; NCTM 1991). The Standards recommended a shift from teacher as lecturer and student as passive listener to the concept of a classroom community. In these classrooms, students would actively participate in the instruction by using logic and mathematical facts to solve problems, and they would employ mathematical reasoning and conjecture in the application of those problems (NCTM, 1991). In 2000, NCTM (NCTM, 2000) published a revised set of standards which address criticisms of the initial version and are more strongly grounded in the cognitive development literature. These revised Standards provide specific suggestions for mathematics instruction in middle school. Specifically, mathematics instruction should emphasize both procedural and conceptual understanding. Individuals should learn not only how to solve the mathematics problems, but why their solution strategy works. This notion of a mathematics classroom is a dramatic shift from the traditional one in which students memorize formulas to solve the problems.

One way for teachers to establish a standards based classroom is to employ the play of mathematically-rich games as a class activity. These games place the responsibility for learning squarely on the students by encouraging them to employ mathematical strategies to succeed. The game playing activity involves the reinforcement of previously learned mathematical concepts and procedures and the guided discovery of new ones. The teacher becomes an observer, a coach, and a facilitator rather than a knowledge keeper who simply supplies knowledge to students (Bauersfeld, 1995). Mathematically-rich games aid student learning because they are contextually meaningful mathematical tasks (Ainley, 1990) that promote mathematical understanding, stimulate classroom discourse, and motivate students to learn.

Mathematical Understanding

Carpenter and Lehrer (1999) defined the five mental processes that stimulate mathematical understanding. Students can gain mathematical understanding when they construct

relationships by taking meaning from the way current concepts are related to other mathematical concepts or previous mathematical experiences. Mathematical understanding also occurs when students extend and apply mathematical knowledge by creating rich, integrated knowledge structures for mathematical concepts. Moreover, individuals who reflect about their mathematical experiences regarding the concepts are able to apply that knowledge to solving unfamiliar problems and gain mathematical understanding. Fourth, when working in a mathematics community, students can gain mathematical understanding by articulating what they know about the mathematical concept in their own words. Finally, when students construct knowledge of the concepts, they gain understanding because they make mathematical knowledge their own. Mathematically-rich games stimulate these forms of mental processes in students when they play the games.

Task Design

Mathematically rich games can be useful in the classroom because they engage the student in meaningful tasks that promote logical, analytical and creative thinking (Ainley, 1990). The authors of the 1991 NCTM Standards document communicated the importance of tasks by choosing as the first standard "Worthwhile Mathematical Tasks" (NCTM, 1991). This standard provides teachers with guidelines on which to base tasks. For instance, tasks in a standards-based classroom should engage students intellectually, develop mathematical understanding and skills, stimulate students to make connections to previous mathematical experiences, require that student engage in problem solving in order to solve the task, stimulate the students to communicate about the task, and represent mathematics as an ongoing human activity by placing the task in a relevant context (NCTM, 1991).

In the report of their work for the QUASAR project, Stein, Smith, Henningsen, and Silver (2000) emphasized the importance of task design in stimulating mathematical understanding in their students. In general, then recommend the frequent use of high cognitive demand tasks to help students gain a greater understanding of the mathematics underlying the task.

Mathematically-rich games are high cognitive tasks that make sense to the students (van Oers, 1996). By trying to find winning strategies, students learn problem-solving strategies such as working backwards, looking for patterns, and considering all possible scenarios (Ainley 1990). In addition, they discover the role of generalizing, conjecture, and posing problems in the problem-solving process (Arcavi, 2002; Mason, Burton, & Stacey, 1985). Moreover, while playing the game, students communicate about the mathematics with each other and the teacher.

Classroom Environment and Discourse

Reinhart (2000) recommends that teachers require everyone to participate. Participation requires the student to engage in the learning process. When playing mathematically-rich games, students, who are motivated to find a winning strategy (Quinn, Koca, & Weening, 1999; Civil, 2002), connect to mathematical investigations and problem solving (Skovsmose, 2002) through discourse with each other and the teacher.

Heaton (2000) describes this discourse as a dance in which the teacher serves as the choreographer, the dancer, the stage manager, and the set designer. Reinhart (2000) would likely say that the teacher is also the audience. To establish a healthy classroom discourse, Reinhart requires students in his classes to ask the questions to each other if they cannot contribute to the discussion and to ask the class for help when it is needed. In this way, students build knowledge by talking to each other about the mathematics while the teacher, who is no longer the source of knowledge, functions as a facilitator.

Mathematically-rich games provide a meaningful context for the students because the outcome, winning or losing, can be important to them (Ainley, 1990). This motivation to succeed (Kamii and Anderson, 2003) stimulates students to learn the material by connecting it to previous experiences, both inside and outside the classroom. Students do not come to the classroom as empty vessels ready to be filled (Piaget, 1954). Rather, they bring with them a rich history of experiences with mathematics, both inside and outside of school. Teachers setting mathematics in context can lead to discoveries that promote confidence in the students (Boaler, 1993). Fasheh (1999) claims that providing a context for mathematics makes it meaningful. By succeeding in the game and seeing this human side of mathematics, students begin to see the role that they can play in the mathematics universe, thereby gaining confidence in themselves.

Sample Game – The Mod Game

The Mod Game is a Nim-Like game in which children count down numbers, according to predetermined rules, to be the first one to reach zero. As children count down, they search for patterns to discover winning strategies. Teachers help students determine winning and losing numbers. In this game, teachers present to their students a variety of concepts, depending on the grade and ability level of their students. Some of the more common concepts include pattern recognition and using counting strategies to multiply and divide numbers.

To play the mod game in the classroom, divide the class into two teams. The object of the game is to be the team that reaches zero. To play the game, one team member will select a whole number as the starting value for the game. Then, the other team can subtract either 1 or 2 from that given number, creating a new number that is closer to zero. The teams take turns subtracting either 1 or 2 until one team wins by reaching zero. Variations of this game include changing the winning rules so that the team that reaches zero loses and varying the numbers that can be subtracted during a turn.

One objective of The Mod Game is for students to recognize the patterns that are characteristic of perfect play. Teachers will know that the students understand the patterns associated with this game by examining the numbers that the players choose and the method of play. Individuals who understand these patterns will choose variations that will ensure that they win the game and will play perfect strategy throughout the game. Students have obtained true mastery of the game when they know who will win based on the starting point and variation of the game.

Results

In the 2006-07 school year one of the authors (DSB) was a weekly volunteer in the two fifth grade classes at Lakewood Elementary School in Durham, NC. On Fridays he would conduct a 45 minute session in one classroom and then move next door for a 45 minute session in the other classroom. A typical session involved dividing the class into teams to play math games aligned to the curriculum. Often the session would end with two questions for the students: 1) "Did you have fun?"; and 2) "Did you learn something?". The answer was always "yes" to both questions.

Although the students' answers are sufficient evidence that they enjoyed the game-playing experience, they are insufficient as evidence of educational progress. For that evidence we turn to the North Carolina End-Of-Grade tests in mathematics and in particular to the expected growth criterion established as part of the ABCs accountability program.
As can be seen from the chart in Figure 1, Lakewood must contend with demographic challenges. Compared to the general population of students in the Durham Public School system, Lakewood has a substantially higher level of students on free and reduced price lunch, English language learners, and exceptional children. It is important, therefore, when making comparisons of educational effectiveness between Lakewood and elsewhere, that the comparisons take cognizance of these challenges. In other words apples should be compared to apples.

The ABCs expected growth criterion accomplishes this apples-to-apples comparison by calculating for each individual student an expectation derived from results in a standard-setting population of students who had performed similarly in previous state assessments. For the fifth grade students at Lakewood to perform on a par level with fifth graders in the standard-setting population, 50% would have to achieve their individually determined expected growth levels. As can be seen from Figure 2, they did outstandingly better than that: 87.5% achieved expected growth or better.

Conclusion

The inclusion of team play in mathematically-rich games as an instructional component of the mathematics curriculum in elementary school is consistent with the pedagogical recommendations of the National Council of Teachers of Mathematics. The implementation of this approach at Lakewood Elementary School in Durham, NC shows great promise, but further research is needed to provide a conclusive demonstration of efficacy.

Figure 1

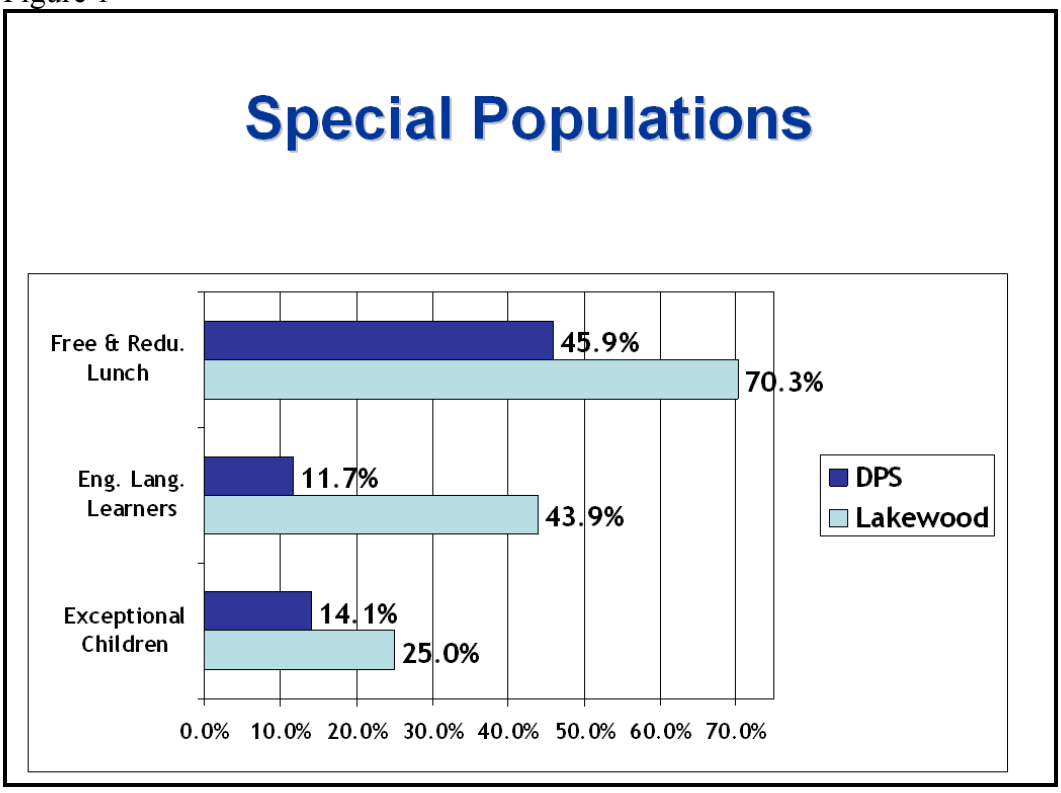
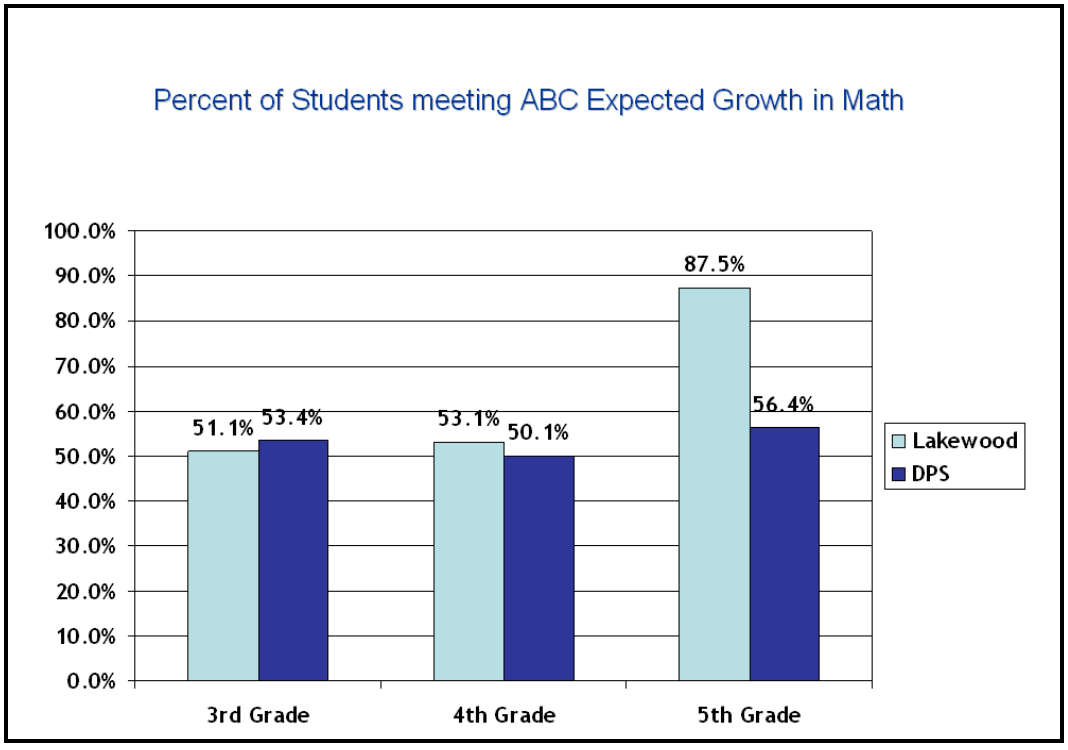


Figure 2



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